1-2 Order of Operations and Evaluating Expressions

Common Core State Standards

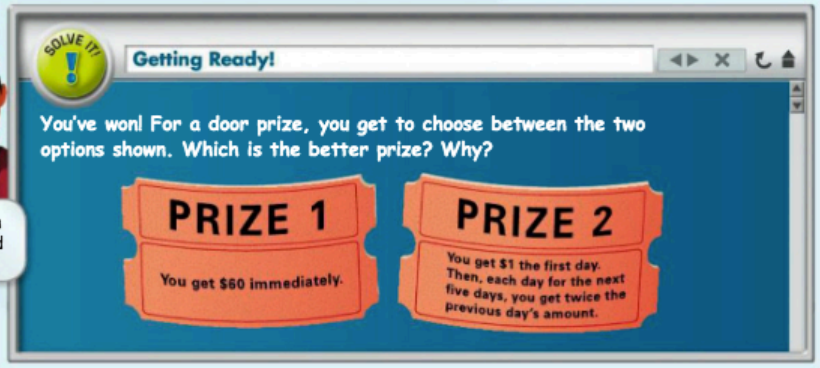
A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MP 1, MP 3, MP 4, MP 6, MP 8

Objectives To simplify expressions involving exponents
To use the order of operations to evaluate expressions

What is your plan for making a good choice?

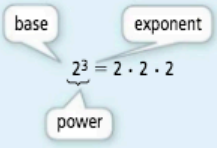
MATHEMATICAL PRACTICES



Essential Understanding You can use *powers* to shorten how you represent repeated multiplication, such as $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

Lesson Vocabulary
- power
- exponent
- base
- simplify
- evaluate

A **power** has two parts, a *base* and an *exponent*. The **exponent** tells you how many times to use the **base** as a factor. You read the power 2^3 as "two to the third power" or "two cubed." You read 5^2 as "five to the second power" or "five squared."



You **simplify** a numerical expression when you replace it with its single numerical value. For example, the simplest form of $2 \cdot 8$ is 16. To simplify a power, you replace it with its simplest name.

Problem 1 Simplifying Powers

What is the simplified form of the expression?

Think
What does the exponent indicate?
It shows the number of times you use the base as a factor.

A 10^7

$10^7 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
$= 10{,}000{,}000$

B $(0.2)^5$

$(0.2)^5 = 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2$
$= 0.00032$

Got It? **1.** What is the simplified form of each expression?

a. 3^4 **b.** $\left(\frac{2}{3}\right)^3$ **c.** $(0.5)^3$

Essential Understanding When simplifying an expression, you need to perform operations in the correct order.

You might think about simplifying the expression $2 + 3 \times 5$ in two ways:

Add first.

$2 + 3 \times 5 = 5 \times 5 = 25$ ✗

Multiply first.

$2 + 3 \times 5 = 2 + 15 = 17$ ✔

Both results may seem sensible, but only the second result is considered correct. This is because the second way uses the order of operations that mathematicians have agreed to follow. Always use the following order of operations:

take note

Key Concept Order of Operations

1. Perform any operation(s) inside grouping symbols, such as parentheses ( ) and brackets [ ]. A fraction bar also acts as a grouping symbol.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Problem 2 Simplifying a Numerical Expression

What is the simplified form of each expression?

A $(6 - 2)^3 \div 2$

$(6 - 2)^3 \div 2 = 4^3 \div 2$ Subtract inside parentheses.

$= 64 \div 2$ Simplify the power.

$= 32$ Divide.

B $\frac{2^4 - 1}{5}$

$\frac{2^4 - 1}{5} = \frac{16 - 1}{5}$ Simplify the power.

$= \frac{15}{5}$ Subtract.

$= 3$ Divide.

Think

How do you simplify an expression that contains a fraction?
You start by simplifying the numerator and denominator. Then you divide the numerator by the denominator.

Got It? 2. What is the simplified form of each expression?

a. $5 \cdot 7 - 4^2 \div 2$

b. $12 - 25 \div 5$

c. $\frac{4 + 3^4}{7 - 2}$

d. **Reasoning** How does a fraction bar act as a grouping symbol? Explain.

When two or more variables, or a number and variables, are written together, treat them as if they were within parentheses. So $4xy$ is equivalent to $(4x)y$, and $xy^2 = (xy^2)$. You **evaluate** an algebraic expression by replacing each variable with a given number. Then simplify the expression using the order of operations.

Problem 3 Evaluating Algebraic Expressions

What is the value of the expression for $x = 5$ and $y = 2$?

Plan

How is this Problem like ones you've seen before?
You begin by substituting numbers for the variables. After substituting, you have numerical expressions just like the ones in Problem 2.

A $x^2 + x - 12 \div y^2$

$x^2 + x - 12 \div y^2 = 5^2 + 5 - 12 \div 2^2$ Substitute 5 for x and 2 for y.

$= 25 + 5 - 12 \div 4$ Simplify powers.

$= 25 + 5 - 3$ Divide.

$= 27$ Add and subtract from left to right.

B $(xy)^2 \div (xy)$

$(xy)^2 \div xy = (5 \cdot 2)^2 \div (5 \cdot 2)$ Substitute 5 for x and 2 for y.

$= 10^2 \div 10$ Multiply inside parentheses.

$= 100 \div 10$ Simplify the power.

$= 10$ Divide.

Got It? **3.** What is the value of each expression when $a = 3$ and $b = 4$ in parts (a)–(b)?

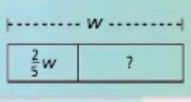
a. $3b - a^2$ **b.** $2b^2 - 7a$

Problem 4 Evaluating a Real-World Expression

Banking **What is an expression for the spending money you have left after depositing $\frac{2}{5}$ of your wages in savings? Evaluate the expression for weekly wages of $40, $50, $75, and $100.**

Think

How can a model help you write the expression?
The model shows that the spending money equals your wages w minus the amount you save: $\frac{2}{5}w$.



Know
- Savings equals $\frac{2}{5}$ of wages.
- Various weekly wages

Need
- Expression for spending money
- Amount of spending money for various weekly wages

Plan
Write an algebraic expression and evaluate it for each amount of weekly wages. Use a table to organize your results.

Relate spending money equals wages minus $\frac{2}{5}$ of wages

Define Let w = your wages.

Write $w - \frac{2}{5} \cdot w$

The expression $w - \frac{2}{5} \cdot w$ represents the amount of money you have left after depositing $\frac{2}{5}$ of your wages in savings.

Spending Money

Wages (w)	$w - \frac{2}{5}w$	Total Spending Money ($)
40	$40 - \frac{2}{5}(40)$	24
50	$50 - \frac{2}{5}(50)$	30
75	$75 - \frac{2}{5}(75)$	45
100	$100 - \frac{2}{5}(100)$	60

Got It? 4. The shipping cost for an order at an online store is $\frac{1}{10}$ the cost of the items you order. What is an expression for the total cost of a given order? What are the total costs for orders of \$43, \$79, \$95, and \$103?

Lesson Check

Do you know HOW?

What is the simplified form of each expression?

1. 5^2
2. 2^3
3. $\left(\frac{3}{4}\right)^2$

Evaluate each expression for $x = 3$ and $y = 4$.

4. $x^2 + 2(x + y)$
5. $(xy)^3$
6. $4x^2 - 3xy$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** Identify the exponent and the base in 4^3.
8. **Error Analysis** A student simplifies an expression as shown below. Find the error and simplify the expression correctly.

$$\begin{aligned} 23 - 8 \cdot 2 + 3^2 &= 23 - 8 \cdot 2 + 9 \\ &= 15 \cdot 2 + 9 \\ &= 30 + 9 \\ &= 39 \quad \text{✗} \end{aligned}$$

1-2 Order of Operations and Evaluating Expressions

Vocabulary

Review

To *simplify* a numerical expression means to replace it with its single numerical value. Circle the *simplified form* of each expression.

1. $2 \cdot 3 \cdot 4$

$4 \cdot 3 \cdot 2$	$6 \cdot 4$	9	24

2. $\frac{1}{2} \cdot 36$

$36 \cdot \frac{1}{2}$	12	18	$36\frac{1}{2}$

3. $16 - 4 + 7$

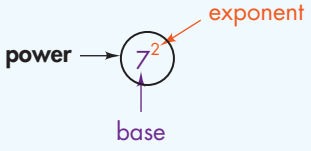
$16 - 7 + 4$	5	10	19

Vocabulary Builder

power (noun) POW er

Related Words: base, exponent

Definition: A **power** is a number that can be expressed using a base and an exponent.



Main Idea: **Powers** provide a shorthand way for showing repeated multiplication.

Example: The diagram above shows a **power**, its *base*, and its *exponent*. You can read the expression as, "seven to the second power."

Use Your Vocabulary

4. Circle the expression that shows a base of 7 and an exponent of 3.

3^7	$7(3)$	7^3	$\frac{3}{7}$

5. Underline the correct word to complete the sentence.

$A(n)$ exponent / power is a number that can be expressed using a base and an exponent.

6. For each expression, underline the base, circle the exponent, and draw a box around the power.

2^5	4^x	m^7	w^z

Problem 1 Simplifying Powers

Got It? What is the simplified form of 3^4?

7. Follow the steps to find the simplified form of the expression.

1 Identify the base and the exponent in the expression 3^4.

base: _____ exponent: _____

2 Expand the expression to show the repeated multiplication indicated by the exponent.

$3^4 =$ _____ $\cdot$ _____ $\cdot$ _____ $\cdot$ _____

3 Write the simplified form of the expression 3^4.

$3^4 =$ _____

take note

Key Concept Order of Operations

1. Perform any operation(s) inside grouping symbols, such as parentheses ( ) and brackets [ ]. A fraction bar also acts as a grouping symbol.
2. Simplify powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Problem 2 Simplifying a Numerical Expression

Got It? What is the simplified form of $5 \cdot 7 - 4^2 \div 2$?

8. Circle the part of the expression that you should simplify first.

$5 \cdot 7 - 4^2 \div 2$

9. Without simplifying the expression, explain how you know that subtraction will be the last operation.

_____

_____

10. Simplify $5 \cdot 7 - 4^2 \div 2$. Show and justify each step.

Problem 3 Evaluating Algebraic Expressions

Got It? What is the value of the expression when $a = 3$ and $b = 4$?

$3b - a^2$

11. $3b - a^2 = 3 \cdot$ ☐ $-$ ☐^{2} Substitute 3 for a and 4 for b.

12. $=$ ☐ $-$ ☐^{2} Multiply.

13. $=$ ☐ $-$ ☐ Simplify the power.

14. $=$ ☐ Subtract.

Problem 4 Evaluating a Real-World Expression

Got It? The shipping cost for an order at an online store is $\frac{1}{10}$ the cost of the items you order. What is an expression for the total cost of a given order? What are the total costs for orders of \$43, \$79, \$95, and \$103?

15. Complete the model.

Relate total cost of an order is the cost of the items plus the shipping costs: $\frac{1}{10} \cdot$ the cost of the items

Define Let $c =$ the cost of the items.

Write total cost $=$ ☐ $+$ $\frac{\square}{\square} \cdot$ ☐

16. Use the model to complete the table for each value of c.

Cost of Items	Shipping Cost	Total Cost of Order
\$43	$\frac{1}{10} \cdot \$43 = \$4.30$	$\$43 + \$4.30 = \$$ ☐
\$79	$\frac{1}{10} \cdot \$79 = \$$ ☐	$\$79 + \$7.90 = \$$ ☐
\$95	$\frac{1}{10} \cdot \$95 = \$$ ☐	$\$95 + \$$ ☐ $= \$104.50$
\$103	$\frac{1}{10} \cdot \$103 = \$$ ☐	$\$103 + \$$ ☐ $= \$$ ☐

Lesson Check • Do you UNDERSTAND?

Error Analysis **A student simplifies an expression as shown below. Find the error and simplify the expression correctly.**

$$23 - 8 \cdot 2 + 3^2 = 23 - 8 \cdot 2 + 9$$
$$= 15 \cdot 2 + 9$$
$$= 30 + 9$$
$$= 39$$

17. What operation did the student do first? Is this correct? Explain.

18. What operation did the student do next? Is this correct? Explain.

19. Now simplify the expression $23 - 8 \cdot 2 + 3^2$ correctly.

Math Success

Check off the vocabulary words that you understand.

☐ power ☐ exponent ☐ base ☐ simplify ☐ evaluate

Rate how well you can *evaluate expressions using the Order of Operations.*

Need to review 0 2 4 6 8 10 Now I get it!

Name_____ Class_____ Date_____

1-2 Practice

Simplify each expression.

1. 4^2

2. 5^3

3. 1^{16}

4. $\left(\frac{5}{6}\right)^2$

5. $(1 + 3)^2$

6. $(0.1)^3$

7. $5 + 3(2)$

8. $\left(\frac{16}{2}\right) - 4(5)$

9. $4^4(5) + 3(11)$

10. $17(2) - 4^2$

11. $\left(\frac{20}{5}\right)^3 - 10(3)^2$

12. $\left(\frac{27-12}{8-3}\right)^3$

13. $(4(5))^3$

14. $2^5 - 4^2 \div 2^2$

15. $\left(\frac{3(6)}{17-5}\right)^4$

Evaluate each expression for $s = 2$ and $t = 5$.

16. $s + 6$

17. $5 - t$

18. $11.5 + s^2$

19. $\frac{s^4}{4} - 17$

20. $3(t)^3 + 10$

21. $s^3 + t^2$

22. $-4(s)^2 + t^3 \div 5$

23. $\left(\frac{s+2}{5t^2}\right)^2$

24. $\left(\frac{3s(3)}{11-5(t)}\right)^2$

25. Every weekend, Morgan buys interesting clothes at her local thrift store and then resells them on an auction website. If she brings \$150.00 and spends s, write an expression for how much change she has. Evaluate your expression for s = \$27.13 and s = \$55.14.

Name_____ Class _____ Date _____

1-2 Practice (continued)

26. A bike rider is traveling at a speed of 15 feet per second. Write an expression for the distance the rider has traveled after s seconds. Make a table that records the distance for 3.0, 5.8, 11.1, and 14.0 seconds.

Simplify each expression.

27. $4[(12 + 5) - 4^4]$

28. $3[(4 - 6)^2 + 7]^2$

29. $2.5\left[13 - \left(\frac{36}{6}\right)^2\right]$

30. $[(48 \div 8)^3 - 7]^3$

31. $\left(\frac{4(-4)(3)}{11 - 5(1)}\right)^3$

32. $4[11 - (55 - 3^5) \div 3]$

33. a. If the tax that you pay when you purchase an item is 12% of the sale price, write an expression that gives the tax on the item with a price p. Write another expression that gives the total price of the item, including tax.

b. What operations are involved in the expressions you wrote?

c. Determine the total price, including tax, of an item that costs $75.

d. Explain how the order of operations helped you solve this problem.

34. The cost to rent a hall for school functions is $60 per hour. Write an expression for the cost of renting the hall for h hours. Make a table to find how much it will cost to rent the hall for 2, 6, 8, and 10 hours.

Evaluate each expression for the given values of the variables.

35. $4(c + 5) - f^4$; $c = -1, f = 4$

36. $-3[(w - 6)^2 + x]^2$; $w = 5, x = 6$

37. $3.5\left[h^3 - \left(\frac{3j}{6}\right)^2\right]$; $h = 3, j = -4$

38. $x[y^2 - (55 - y^5) \div 3]$; $x = -6, y = 6$